A Technique for Predicting Skin-Effect in Digital Transmission

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Here is a formula that can be used to accurately compute the transmission line response to an arbitrary waveshape and thus simplify and render a more precise evaluation of digital transmission system performance

One of the problems in designing a data transmission system is determining the relationship of specified attenuation/100 feet of cable at a given frequency to the digital or pulse response of the transmission line.

The attenuation characteristic is related to the skin-effect response of transmission lines and is a result of the physical phenomena of self-inductance. As the frequency of the transmitted energy is increased, the ac resistance of the wire increases. This resistance is greatest in the central core of the conductor and decreases toward the surface. The result is that most current carriers are crowded into the shell or "skin" at the conductor surface. The higher the frequency, the thinner the shell. In the same manner, for a fixed frequency the ac resistance of a conductor increases inversely as the square of the radius of the wire.

The phenomenon is quite well known and all transmission line manufacturers specify the information generally as attenuation in dB/100 feet of cable at one or more frequencies. The problem is, how does this attenuation characteristic distort the waveshape when the driving function is high frequency digital information?

Some work has been done in this area.[1,2] In fact, the transmission line response to an idealized step function is often referred to as the "familiar" co-error function response. A normalized co-error function response is compared with a normalized exponential function in Fig. 1.

The work done by Wiggington and Nahman[1,2] extended the theory to allow the prediction of the transmission line response to an input excitation that is approximated by straight line segments. Their method requires the use of graphs or a complex integration process.

My work and approach can be considered as a further extension of the previous work in the sense that it reduces the problem of response predication to a relatively minor computer programming effort. An addi-

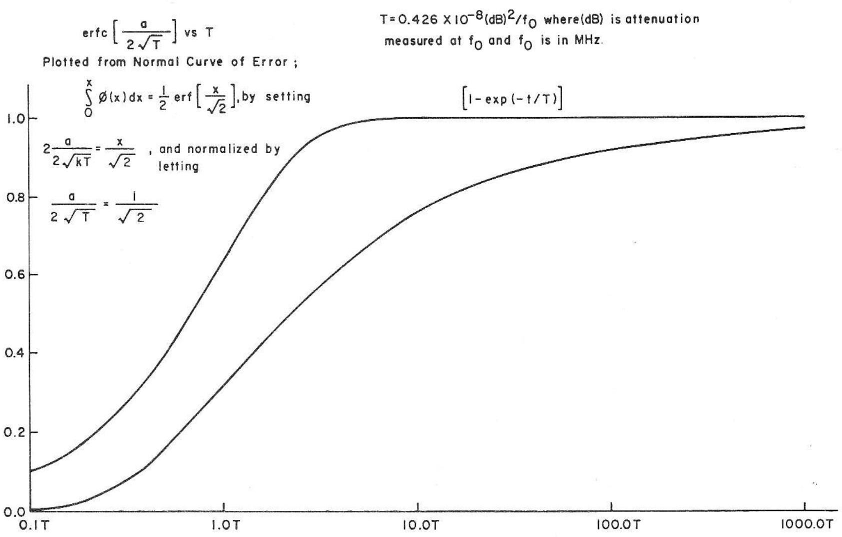

Fig. 1 Comparison of a normalized co-error function response with a normalized exponential function

tional refinement is the inclusion of terminal reflections caused by a resistive terminal impedance mismatch.

Analysis

The circuit model used to develop the network equations is shown in Fig. 2. The driving function Vi(t) is generalized by segmentizing the waveshape into a finite number of straight lines with various lengths and slopes (Fig. 3).

Using Laplace transform techniques to solve the basic telegraphics equation and inserting the transform of the generalized driving function yields the following expression for the transmission line response $V(l, s)$:

$$V(l, S) = V_o(S) = \frac{R}{S^2} \sum_{n=0}^{N} U_n(e^{-ST_{n-1}} - e^{-ST_n}) \sum_{N=0}^{2} e^{-(2n+1)\gamma(S)\,l\rho^n} \qquad (1)$$

where N is the number of ramps required to approximate the particular input function desired,

$$R = \frac{2R_L^2}{(R_L + R_{dc} + R_G)\ (R_L + Ro)},$$

$$\left(U_n = \frac{V_n - V_{n-1}}{T_n - T_{n-1}}\right), \text{ and}$$

$$\rho = (\rho_N)\ (\rho_G) = \left(\frac{R_C - Ro}{R_L + Ro}\right) \left(\frac{R_G - Ro}{R_G + Ro}\right) \qquad (2)$$

and Ro is the transmission line impedance.

Some simplfying assumptions and approximations are made regarding the propagation constant $\gamma(S) = [(LS + R)\ (CS + G)]^{1/2}$. First the conductance, G, is assumed to be equal to zero. This is valid in most high quality transmission lines up to frequencies in the 2 to 3 GHz range. The expression for γ is further modified to include the resistive and inductive components dependency on frequency[1]; $\gamma = [(LS + K\sqrt{S})CS]^{1/2}$ where K is a function of the physical dimensions of the transmission line. This equation is expanded using the binomial expansion theorem. The result is an alternating infinite series that is truncated to obtain the following approximate relationship:

$$\gamma = STd + \frac{K\sqrt{S}}{2\ Ro} \qquad (3)$$

where Td is a time delay term.

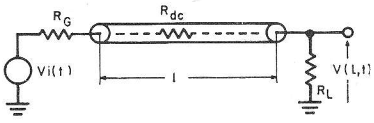

Fig. 2 Transmission line circuit model used in developing the network equations

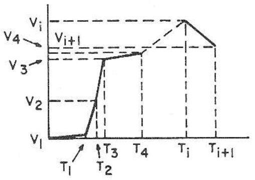

Fig. 3 Construction of single pulses, or trains of pulses

The second summation in Eq. (1) results from the inclusion of the reflection terms ρ_r and ρ_g. In theory this is an infinite summation but only three terms are included in Eq. (1). This was done for three reasons: (1) Each term is multiplied by ρ^n which in most cases of practical interest is a number $<<1$ for $n \geqq 3$; (2) It is known that these reflections do not build up in a lossy system; and (3) Any finite number of terms may be included at the discretion of the user but three terms yield very satisfactory results and simplifies the programming somewhat.

In Eq. (1) there are terms involving e^{-ST_n} and $e^{-(2n+1)STdl}$, which represent time delay. The terms of interest are of the form $e^{-a_n\sqrt{S}}/S^2$. This expression has an inverse[3] as follows:

$$L^{-1}\,[e^{-a_n\sqrt{S}}/S^2] = (t + a_n^2/2)\ \text{erfc}\ (a_n/2\sqrt{t})\ \frac{-a_n\sqrt{t}}{\sqrt{\pi}}\, e^{-a^2/4t} \qquad (4)$$

The factor a_n can be shown to have the following relationship:

$$a_n = \frac{(2n + 1)\ A}{868.6\sqrt{\pi f_o}} \qquad (5)$$

where A is the attenuation in dD/100′ measured at f_o. The value of A is that furnished by the manufacturer.

Designating Eq. (4) as f(t, n) for the output voltage $V_o(t)$, the desired solution can be compactly expressed as:

$$\begin{aligned} V_o(t) = R \sum_{i-1}^{N} \left[\frac{V_i - V_{i-1}}{T_i - T_{i-1}}\right] & \Big\{ [f(t - T_{i-1} - T, 0) - f(t - T_i - T, 0)] \\ & + [f(t - T_{i-1} - 3T, 1) \\ & - f(t - T_i - 3T, 1)]\,\rho \\ & + [f(t - T_{i-1} - 5T, 2) \\ & - f(t - T_i - 5T, 2)]\,\rho^2 \Big\} \end{aligned} \qquad (6)$$

The function f(t, n) as used in Eq. (6) is defined as follows:

$$f(t,n) = \begin{cases} 0, & t \leq 0 \\ f(t, n), & t > 0 \end{cases}$$

The values of V_i and T_i [in Eq. (6)] are supplied by the user. Single pulses or trains of pulses may be constructed (as in Fig. 3) depending on the requirements.

A program was written in FORTRAN language to solve Eq. (6) for a series of incremental values of the argument t. An available print plot routine gave the desired graphic "picture" of the output response. The resulting computer predicted responses showed extremely good correlation with transmission line responses measured on the laboratory test bench.

Summary

The result of this work has been used to predict the performance of a number of parallel transmission line systems with good success. The straight line segmentation of the waveshape allows any realizable driving signal to be approximated including pulse trains. A great advantage is that the effects of positive mismatches and driving signal compensation networks can be "observed" prior to committing to hardware implementation. The formulation, however, does have limitations. It cannot be used to predict responses in a multistub environment; nor is it capable of handling nonlinear loads. It is, however, quite useful in many cases of practical interest and particularly in predicting the skew across a bus.

References

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2. R. L. Wiggington and N. S. Nahman, "Transient Analysis of Coaxial Cables Considering Skin Effect," *Proceedings of the IRE,* 1957, pp 166-174
3. G. E. Roberts and H. Kaufman, *Table of Laplace Transforms,* W. B. Saunders Co, 1966